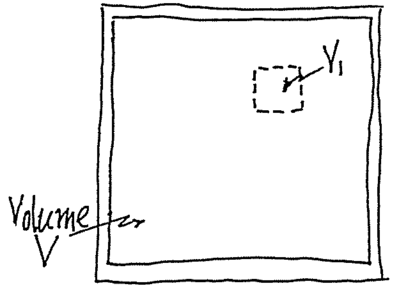
Class Work: Spatial Distribution of Gas Molecules

N molecules in volume V (e.g. $\sim 10^{22}$ in a litre or $\sim 10^{25}$ in m^3)



V_1 = a smaller volume inside V

System is in equilibrium.

Q: How many molecules are there in V_1?
What is the fluctuation in this number?

- Expectations from common sense:

(1) At equilibrium, density of gas is uniform in V

$$\Rightarrow \#\text{ molecules in } V_1 = \left(\frac{V_1}{V}\right) \cdot N$$

(2) As molecules are constantly moving, this number fluctuates.

Q: How could we get at these common sense expectations mathematically, and more?

The power of ignorance!

- We know that a molecule must be somewhere within V.
- Simplest assumption: a molecule is equally likely to be anywhere in V

p = probability that a molecule is in V_1 $= \dfrac{V_1}{V}$

$q = 1 - p$ = probability that a molecule is not in V_1

One-dimensional random walk: p = prob. stepping to the right
q = prob. stepping to the left

For a long walk with $N \gg 1$ steps, what is the prob. $P_N(m)$ that the walker is m steps to the right of the origin (m could be +ve or −ve)? Mean displacement? Standard deviation?

Remarks: Things to learn from the example

- Formal ways of evaluating mean and variance given a distribution
- Vast number of entities (particles) gives sharp & representative Mean
- Vast number of independent entities (particles) gives Gaussian (Normal) distribution

Extensions

- Gaussian distribution is completely characterized by mean & variance
- Central Limit Theorem.